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CALCULATION OF THE AERODYNAMIC INTERFERENCE OF A SYSTEM OF BODIES IN AN IDEAL FLUID

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Description of a method for machine computation of the aerodynamic interference of a system of bodies, which is suitable for solving a wide class of linear problems for which the principle of superposition is applicable. The method is based on the representation of the velocity potential of flows past a system of bodies in the form of a sum of velocity potentials of flows past individual bodies, taking into account a certain desired downwash which varies along the surface of a body and which is dependent on the interference of other bodies in the system. As an example, the problem of unsteady flow of an incompressible fluid past a biplane is solved.

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In the development of theoretical methods for calculating the aerodynamic interference of a system of bodies, individual methods were used for each case. Methods based on the theory of conformal mapping are best. For example, these methods permitted great progress in the hydrodynamic theory of airfoil cascades around which flows a plane steady flow of an ideal fluid (Bibl.1, 2, 3).

Owing to the rapid growth of electronic computer equipment and perfection of the methods of computer mathematics, it has become possible to develop and realize in practice a new method of calculating aerodynamic interference of a system of bodies, suitable for solving a broad class of linear problems for which we use the principle of superposition. The idea of the method is very

* Numbers in the margin indicate pagination in the original foreign text.

simple and amounts to representing the velocity potential of a flow past a system of bodies as the sum of the velocity potentials of the flow past single bodies with consideration of a certain unknown downwash of the flow variable along the body surface and caused by interference of other bodies of the system. The resultant velocity potential of flow past a system of bodies will satisfy all conditions of the problem except the condition of zero flow of bodies newly entering the system. Satisfaction of this condition leads to a system of linear equations for determining the unknown downwash of the flow about each body.

The method for calculating the interference is directly applicable to systems consisting of a finite number of isolated bodies. However, in certain cases solutions for systems with an infinite number of elements can also be derived by this method. For instance, in some papers (Bibl.4), problems of the unsteady flow past plate cascades at a small angle of attack were solved by this method.

The basic principles of the proposed method of calculating the interference of a system of bodies are given in this paper. As an example, the problem of unsteady flow of an incompressible fluid about a biplane is solved.

1. Let us examine a system of N bodies, moving in an ideal fluid in such a manner that the distance between the bodies remains practically unchanged. We will associate the coordinate system x_j, y_j, z_j with the j-th body ($j = 1, 2, \ldots, N$). Furthermore, we will introduce the principal coordinate system x, y, z, which, in particular, can coincide with any of the coordinate systems x_j, y_j, z_j ($j = 1, 2, \ldots, N$). The relation between the coordinates of each point of space is assumed to be given in all coordinate systems.

Let us assume that the flow of the fluid outside each body and the vortex sheet trailing from it, if it is formed at all, is potential and can be de-

scribed by a linear equation for the velocity potential with corresponding linear boundary conditions; for a flow past bodies with circulation, the position of the vortex sheet downstream is assumed to be known. We will assume further that the velocity potentials of flow past each single body are known. The velocity potential of flow past the j-th single body $\varphi_j(x_j, y_j, z_j, t)$ depends linearly on the given normal component of the velocity of motion $v^{(j)}$ of the surface $S_j(x_j', y_j', z_j', t)$ of the j-th body [x_j', y_j', z_j' are the surface coordinates of S_j in the system (x_j, y_j, z_j)].

In investigating unsteady motions of the fluid, we will assume that the motion of the surface of the j-th body is harmonic, with an angular frequency ν.

Then, within the framework of the linear theory, we have /34

$$\varphi_j(x_j, y_j, z_j, t) = \varphi_{0j}(x_j, y_j, z_j) + e^{i\nu t}\Phi_j(x_j, y_j, z_j)$$
$$v^{(j)}(x_j', y_j', z_j', t) = v_0^{(j)}(x_j', y_j', z_j') + v_1^{(j)}(x_j', y_j', z_j'). \qquad (1.1)$$

Here, φ_{0j} is the velocity potential of flow past the j-th body by a steady stream determinable regardless of the unsteady state of flow past the body; Φ_j is a function determining the unsteady part of the velocity potential of flow past the j-th body and depending, in the general case, on φ_{0j}; $v_0^{(j)}$ is the normal component of the steady velocity of motion of the surface of the j-th body; $v_1^{(j)}$ is the amplitude of the normal component of the oscillation velocity of the surface of the j-th body.

The given functions $v_0^{(j)}$, $v_1^{(j)}$ on the surface S_j will be approximated either by Fourier polynomials or by power polynomials, or by the values of $v_0^{(j)}$, $v_1^{(j)}$ at n discrete points of the surface S_j. In both cases, the velocity potential of flow past the j-th body can be presented in the form

$$\varphi_{0j}(x_j, y_j, z_j) = \sum_{r=0}^{n-1} \theta_{0r}^{(j)}\varphi_r^{(j)}; \quad \Phi_j(x_j, y_j, z_j) = \sum_{r=0}^{n-1} \theta_{1r}^{(j)}\Phi_r^{(j)}. \qquad (1.2)$$

Here $\theta_{or}^{(j)}$, $\theta_{1r}^{(j)}$ are coefficients of approximation or the values at n discrete points of the surface of the j-th body of the functions $v_0^{(j)}$ and $v_1^{(j)}$, respectively; $\varpi_r^{(j)}(x_j, y_j, z_j)$, $\Phi_r^{(j)}(x_j, y_j, z_j)$ are known functions not dependent on the law of body motion.

The velocity potential $\varpi(x, y, z)$ of flow past a system of N bodies will be sought in the form of the sum of the velocity potentials of flow past single bodies, whose law of surface motion of each j-th body is determined by the sum of the normal velocity components $v^{(j)}$ and $u^{(j)}$. The function $u^{(j)}$ characterizes the wash of the flow at surface points of the j-th body, induced by the remaining N - 1 bodies moving according to given laws $v^{(m)}$ (m = 1, 2, ..., N; m ≠ j). For the motion of the fluid under consideration, the unknown function $u^{(j)}$ can be presented in the form

$$u^{(j)}(x_j', y_j', z_j', t) = u_0^{(j)}(x_j', y_j', z_j') + e^{i\omega t} u_1^{(j)}(x_j', y_j', z_j') . \qquad (1.3)$$

We will assume that, on the surface S_j, the functions $u_0^{(j)}$, $u_1^{(j)}$ are approximated in the same manner as the functions $v_0^{(j)}$, $v_1^{(j)}$. The coefficients of this approximation $\vartheta_{or}^{(j)}$, $\vartheta_{1r}^{(j)}$ (r = 1, 2, ..., n; j = 1, 2, ..., N) are presently unknown. In conformity with eqs.(1.1) and (1.2), we have

$$\varphi(x_j, y_j, z_j, t) = \sum_{j=1}^{N} \sum_{r=1}^{n-1} [(\vartheta_{or}^{(j)} + \vartheta_{1r}^{(j)}) \varphi_r^{(j)}(x_j, y_j, z_j) + \vartheta_{1r}^{(j)} \varphi_n^{(j)}(x_j, y_j, z_j)] \cdot e^{i\omega t} . \qquad (1.4)$$

Equation (1.4) for $\varpi(x, y, z, t)$ will determine the velocity potential of flow past a system of bodies, if the function ϖ satisfies the conditions of damping of perturbation velocities at an infinite distance from the considered system of bodies, the pressure continuity on passing through the vortex wake, and the conditions of zero flow as well as the Joukowski condition on each body of the system. The conditions at infinity and the Joukowski condition are

satisfied, since the potentials of flow past single bodies satisfy them. The vortex wake conditions are also satisfied automatically, if the position of the vortex wakes downstream changes little as a consequence of interference.

In certain cases of a two-dimensional unsteady flow, the conditions in the vortex wakes caused by unsteady motion of the bodies can be satisfied even in /35 the presence of severe interference between the bodies in a steady flow. For this purpose, we must first solve the problem of the interference of bodies in a steady flow and then in an unsteady flow; when solving the problem on unsteady flow past a single body, we must use the velocity potential from the system of vortices situated along the corresponding streamline of the steady flow past the system of bodies.

It should be mentioned that we only indicate here the theoretical possibility of such a solution, whose practical realization is still quite limited at present.

This leaves the condition of zero flow on each of the considered bodies to be satisfied. Namely, at points of the surface S_j,

$$\frac{\partial \Phi}{\partial n_j} = g^{(j)} \; ; \quad (j = 1, \ldots, N) \tag{1.5}$$

Here, n_j is an exterior normal to S_j. Since, on the surface of the j-th body,

$$\frac{\partial}{\partial n_j} \sum_{r=0}^{n-1} (\ldots) \Phi_r^{(j)} = v_1^{(j)} + u_1^{(j)},$$

then, with consideration of eq.(1.4), the conditions (1.5) can be written in the form (j = 1, ..., N)

$$\ldots + \sum_{k=1}^{N} \sum_{r=0}^{n-1} (\ldots) \ldots (x_k, y_k, z_k) = 0 \tag{1.6}$$

$$\ldots + \sum_{\substack{k=1 \\ (k \neq j)}} \ldots = 0 \tag{1.7}$$

Here, the coordinates x_k, y_k, z_k are connected with the coordinates x'_j, y'_j, z'_j of surface S_j of the j-th body by known relations.

Since the functions $u_0^{(j)}$, $u_1^{(j)}$ are linearly related with the coefficients $\vartheta_{0r}^{(j)}$, $\vartheta_{1r}^{(j)}$ ($j = 1, 2, \ldots, N$; $r = 0, 1, 2, \ldots, n-1$), then, requiring satisfaction of the equalities (1.6) and (1.7) at n points of the surface S_j of the j-th body ($j = 1, 2, \ldots, N$), we obtain two independent systems of linear algebraic equations of the nN order for determining 2nN unknown coefficients $\vartheta_{0r}^{(j)}$, $\vartheta_{1r}^{(j)}$ ($j = 1, 2, \ldots, N$; $r = 0, 1, 2, \ldots, n-1$).

As a result of solving the linear system (1.6), (1.7), the velocity potential of flow past a system of bodies is fully determined. The following facts should be mentioned:

1) Realization of the proposed method of calculating the aerodynamic interference of a system of bodies is possible, as a rule, only by means of an electronic computer.

2) The algorithm of the solution of a broad class of problems in the case under consideration is set up by the standard method, which combines the known solutions of problems of flow past single bodies composing a system.

2. As an example let us examine a two-dimensional steady flow of an incompressible fluid about a biplane formed by two thin wings of low camber. The solution of the corresponding problem in a steady flow is known (Bibl.2, 5).

Let us assume that the airfoils vibrate synchronously at low amplitudes with a frequency ν. The vibration modes of each airfoil may differ. The relative flow, at an infinite distance, has a velocity V directed parallel to the wing chord. Let us denote the wing chords by b_1 and b_2; h will be the distance between them, along the axis of the biplane passing through the middle of the

wing chords; β is the stagger angle of the biplane (Fig.1); k_1, k_2 are the Strouhal numbers in an unsteady flow past the first and second isolated airfoils ($k_j = \nu b_j/2V$; $j = 1, 2$). /36

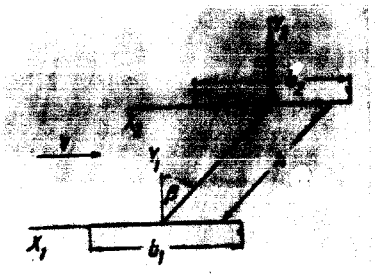

Fig.1

We will relate a system of rectangular coordinates X_j, Y_j ($j = 1, 2$) with each airfoil. The dimensionless coordinates x_1, y_1, and x_2, y_2 of each point are connected by the relations

$$\frac{b_2}{b_1}x_2 = x_1 + 2\varkappa^{-1}\sin\beta$$
$$\frac{b_2}{b_1}y_2 = y_1 - 2\varkappa^{-1}\cos\beta \qquad (2.1)$$
$$(x_j = \ldots \quad Y_j = \ldots \quad \ldots \quad \varkappa = b_1/b_2, \; j = 1.2).$$

The amplitude of the normal velocity component of the j-th airfoil $v_1^{(j)}(x_j)$ and the unknown function $u_1^{(j)}(x_j)$ determining the downwash of the flow will be represented at points of both airfoils by Fourier series of the form

$$v_j(\ldots) = V\left[\frac{\vartheta_0^{(j)}}{2} + \sum_{\ldots}^{\infty} \ldots\right] \ldots V\left[\frac{\vartheta_0^{(j)}}{2} + \sum_{\ldots}^{\infty} \ldots\right], \qquad (2.2)$$
$$(\ldots \quad j = 1.2)$$

The velocity potential $\varphi_j(x_j, y_j, t)$ of a flow past the j-th airfoil which vibrates with the normal velocity component $[v_1^{(j)} + u_1^{(j)}]\exp(i\nu t)$ can be represented as (Bibl.2)

$$\varphi_j(x_j, y_j, t) = e^{i\nu t}\sum_{r=0}^{\infty}(\ldots\vartheta_r^{(j)})\,\Phi_r^{(j)}(x_j, y_j) \quad (j = 1, 2). \qquad (2.3)$$

Here,

$$\Phi_0^{(j)} = \frac{1}{2} c_j V \left[-e^{-\xi_j} \sin \eta_j + e^{ik_j x_j} \int_{\infty}^{x_j} e^{-ik_j u} \frac{\partial g_j^*(u, y_j)}{\partial y_j} du \right]$$

$$\Phi_1^{(j)} = \frac{1}{2} c_j V \left[-\frac{1}{2} e^{-2\xi_j} \sin 2\eta_j - e^{-ik_j x_j} \int_{\infty}^{x_j} e^{-ik_j u} \frac{\partial g_j^*(u, y_j)}{\partial y_j} du \right] \qquad (2.4)$$

$$\Phi_r^{(j)} = \frac{1}{2} c_j V \left[-\frac{1}{r+1} e^{-(r+1)\xi_j} \sin (r+1)\eta_j + \frac{1}{r-1} e^{-(r-1)\xi_j} \sin (r-1)\eta_j \right].$$
$$(r \geqslant 2)$$

The function

$$g_j^* = [1 - C(k_j)] \eta_j^* + C(k_j) \xi_j^* . \qquad (2.5)$$

Here $C(k_j)$ is the Theodorsen mapping function, and the elliptical coordinates ξ_j, η_j are related to the rectangular coordinates x_j, y_j ($j = 1, 2$) by

$$\sin \eta_j = y_j \sinh \xi_j, \qquad \cos \eta_j = x_j \cosh \xi_j$$
$$\sinh^2 \xi_j = -1/2 [1 - x_j^2 - y_j^2] + \sqrt{1/4 (1 - x_j^2 - y_j^2)^2 + y_j^2} . \qquad (2.6)$$

The variables ξ_j^*, η_j^* are determined from the same relations (2.6) provided that x_j is replaced by $u (x_j \leq u < \infty)$.

We note that, in our case, the direction of the normals to the airfoils coincides with the axes y_1, y_2. In the series (2.2), (2.3) we limited ourselves to the first n terms. Then, taking into account eqs.(2.1) - (2.6), the expression (1.7) for determining the unknown coefficients $\vartheta_r^{(j)}$ ($j = 1, 2$; $r = 0, 1, \ldots, n-1$) can be reduced to the following form: /37

$$\frac{1}{2} \vartheta_0^{(1)} + \sum_{r=1}^{n-1} \vartheta_r^{(1)} \cos r \qquad \sum \qquad = 0 ;$$
$$\frac{1}{2} \vartheta_0^{(2)} + \sum_{r=1}^{n-1} \vartheta_r^{(2)} \cos \qquad = 0 . \qquad (2.7)$$

Here, the functions $X_{r,j}(\eta)$ ($j = 1, 2$) are determined by the equalities

$$X_{0j} = X_{1j}^{(0)} + X_{0j}^{(1)}, \qquad X_{1j} = X_{2j}^{(0)} - X_{0j}^{(1)}, \qquad X_{rj} = X_{r+1,\,j}^{(0)} - X_{r-1,\,j}^{(0)} \quad \text{for } r \geqslant 2;$$

$$X_{0j}^{(1)}(\eta) = -\frac{\partial g_j^*}{\partial x_j} - ik_j e^{ik_j x_j} \int_{\infty}^{x_j} e^{-ik_j u} \frac{\partial g_j^*(u,\, y_j)}{\partial u}\, du;$$

$$X_{rj}^{(0)}(\eta) = \frac{\exp(-r\xi_j)}{\xi_j - \cos\eta_j} [\xi_j \sin\eta_j \sin r\eta_j - \xi_j \cos\eta_j \cos r\eta_j] \quad (r = 1, 2, \ldots).$$

The coordinates ξ_2, η_2 on which depends the function $X_{r2}(\eta_1)$, are related to the coordinate η_1 by eqs.(2.1), (2.6) in which we set $y_1 = 0$, $x_1 = \cos\eta_1$, while the coordinates ξ_1, η_1 entering into the expression for the function $X_{r1}(\eta_2)$ are determined from eqs.(2.1), (2.6) for $y_2 = 0$, $x_2 = \cos\eta_2$.

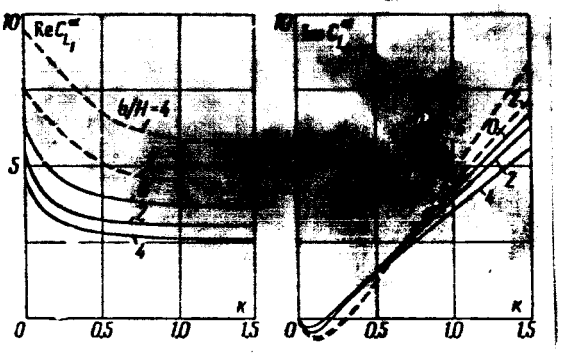

Fig.2

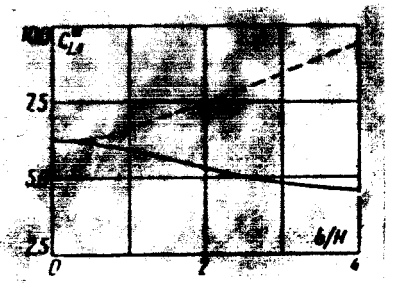

Fig.3

Stipulating that eqs.(2.7) must be satisfied at n points on each airfoil, we will obtain a system of simultaneous algebraic equations of the 2n order for determining the unknown coefficients $\vartheta_r^{(1)}$, $\vartheta_r^{(2)}$. Solving this system and

substituting the found coefficients $\vartheta_r^{(1)}$, $\vartheta_r^{(2)}$ ($r = 0, 1, \ldots, n-1$) into eq.(1.4), we derive the expression for the velocity potential of an unsteady flow past a biplane (in this case, $v_0^{(j)} = 0$, $u_0^{(j)} = 0$; $j = 1, 2$).

Since the wings of the biplane are thin and the velocity potential of the flow past the k-th isolated airfoil is continuous at the points of the j-th airfoil ($j \neq k$), the pressure drop acting on the pressure and suction sides of the j-th airfoil depends only on the potential φ_j determined by eq.(2.3). In conformity with this, the aerodynamic force and the moment acting on the j-th airfoil can be represented in the form (Bibl.2)

$$P^{(j)} = -2\pi\rho_\infty V_\infty^2 e^{i\nu t} b_j C(k_j)(\theta_0^{(j)} + \theta_0^{(j)}_1 - \theta_1^{(j)} - \theta_1^{(j)}) + \quad (2.8)$$

$$+ \quad (2.9)$$

The moment $M^{(j)}$ is calculated relative to the center of the profile with a positive direction of rotation from the axis X_j to Y_j (Fig.1).

For different laws of motion of airfoils in the usual linear setup, it /38 is convenient to calculate the aerodynamic forces and moments acting on each airfoil when only one of the airfoils vibrates and the other remains fixed. Then, the combination of these forces and moments permits calculation of the forces and moments acting on the wings of a biplane.

Restricting ourselves to the case of torsional vibrations of the biplane wings, we will represent the expressions for $P^{(j)}$ and $M^{(j)}$ in the form ($j = 1,2$)

(2.10)
(2.11)

Here, α_1, α_2 are the angles of rotation about the center chord of the first and second airfoils; the coefficients $l_\alpha^{(j)}$, $m_{\alpha k}^{(j)}$ determine the forces and mo-

ments acting on the j-th airfoil during vibrations according to the corresponding law of the k-th airfoil (j, k = 1, 2).

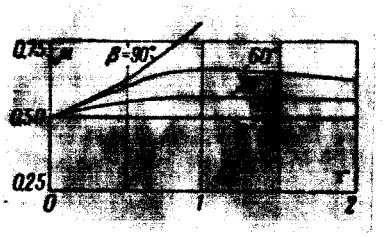

Fig.4

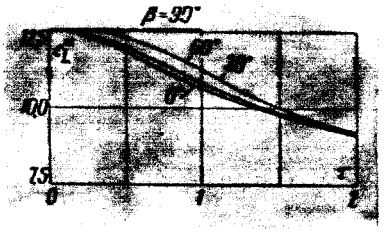

Fig.5

At $b_1 = b_2$ and $\alpha_1 = \alpha$, $\alpha_2 = \alpha e^{i\psi}$ (ψ is the phase shift of vibrations of the second airfoil relative to the first), the expressions for $P^{(j)}$, $M^{(j)}$ can be represented in the form

$$\left.\begin{aligned} P^{(j)} &= {}^1/_2\rho V^2 b C_{Lj}{}^{\alpha}\alpha, \quad C_{Lj}{}^{\alpha} = l_{\alpha_1}{}^{(j)} + e^{i\psi} l_{\alpha_2}{}^{(j)} \\ M^{(j)} &= {}^1/_2\rho V^2 b^2 C_{mj}{}^{\alpha}\alpha \\ C_{mj}{}^{\alpha} &= m_{\alpha_1}{}^{(j)} + e^{i\psi} m_{\alpha_2}{}^{(j)} \end{aligned}\right| \qquad (2.12)$$

As an example, we calculated the aerodynamic forces acting on the wings of a biplane which were executing torsional vibrations about the center chord. The calculations were carried out on an electronic computer. Equations (2.7) were satisfied at five points of each airfoil, whose coordinates were $x_m = \cos(m\pi/4)$ ($m = 0, \ldots, 4$). For the Strouhal number $k \neq 0$, the coefficients $\vartheta_r^{(1)}$, $\vartheta_r^{(2)}$ proved to be complex.

We note that a flow past a biplane without stagger, in the presence of

cophased vibrations of the airfoils, corresponds to the motion of a vibrating airfoil close to the free surface of a fluid; in the presence of out-of-phase vibrations of the airfoils, the flow corresponds to a motion close to the ground. In this case, the relative distance $\tau = b_1/h$ is correlated with the distance H from the profile to the corresponding surface by the relation $b/H = 2\tau$. In the case of $k = 0$, these results correspond to a steady flow past the biplane wings.

Figure 2 shows the results of calculating the aerodynamic forces acting on the airfoil close to the free surface of the fluid (solid curve) and close to the surface of the ground (dashed curve), as a function of the Strouhal number $k = \nu b/2V$.

Figure 3 shows similar results for the steady motion of the airfoil close to the interface of two media. It is of interest to define the share of participation of each airfoil in the formation of the total aerodynamic load, as a function of the stagger β and the relative distance τ. Figure 4 gives the value of $\mu = P_1/P$, where P_1 is the lift acting on the first airfoil and P is the total lift of the biplane. We note that, for $\beta = 90^\circ$, we have a tandem-type biplane for which the values of τ in the interval $0 \leq \tau < 1$ have a physical sense.

Figure 5 gives the values of the total lift of the biplane, C_L^α. The results shown in Figs.4 and 5, for the case of a tandem biplane, coincide with the corresponding results obtained by L.I.Sedov (Bibl.2).

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